Period 1, March 3, 2025

$$\int_0^2 \frac{e^{2x}}{e^{2x}+1}\,dx$$

$u = e^{2x}+1$

$du = 2e^{2x}\,dx$

$\frac{du}{2e^{2x}} = dx$

$$\int_2^{e^4+1} \frac{\cancel{e^{2x}}}{u} \cdot \frac{du}{2\cancel{e^{2x}}} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$u = e^{2\cdot 2}+1 = e^4+1$

$u = e^{2\cdot 0}+1 = 1+1$

$u = e^{2x}+1 \Rightarrow u = e^{L}+1$

$L = 2x$

$\frac{dL}{dx} = 2$

$\frac{du}{dL} = e^{L}+0$

$\frac{\cancel{dL}}{dL} \cdot \frac{du}{\cancel{dL}} = 2\cdot e^{L} = 2\cdot e^{2x}$

$\frac{1}{2}\ln|e^{2x}+1| \Big|_0^2$

$\frac{1}{2}\ln|e^4+1| - \frac{1}{2}\ln|\underbrace{e^0}_{=1}+1|$

$\frac{1}{2}\ln|e^4+1| - \frac{1}{2}\ln|\underbrace{1+1}_{=2}|$

$\frac{1}{2}\ln|e^4+1| - \frac{1}{2}\ln 2$

$\frac{1}{2}\left[\ln|e^4+1| - \ln 2\right]$

$\frac{1}{2}\cdot\ln\frac{e^4+1}{2}$

$$\int_1^3 \frac{e^{3x}}{e^{3x}-1}\,dx = \int \frac{\cancel{e^{3x}}}{u}\cdot\frac{du}{3\cancel{e^{3x}}} = \frac{1}{3}\int_{e^3-1}^{e^9-1}\frac{du}{u} = \frac{1}{3}\ln|u|$$

$u = e^{3x}-1$

$du = 3e^{3x}\,dx$

$\frac{du}{3e^{3x}} = dx$

$u = e^{3\cdot 3}-1 = e^9-1$

$u = e^{3\cdot 1}-1 = e^3-1$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$e^9-1 = (e^3-1)(e^{2\cdot 3}+1\cdot e^3+1^2)$

$\frac{1}{3}\ln|u| = \frac{1}{3}\ln|e^{3x}-1| \Big|_1^3$

$\frac{1}{3}\ln|e^{3\cdot 3}-1| - \frac{1}{3}\ln|e^{3\cdot 1}-1|$

$\frac{1}{3}\ln e^9-1 - \frac{1}{3}\ln|e^3-1|$

$\frac{1}{3}\left[\ln\frac{e^9-1}{e^3-1}\right] = \frac{1}{3}\left[\ln e^6+e^3+1\right]$

$\frac{1}{3}\left[\ln\frac{\cancel{(e^3-1)}(e^6+e^3+1)}{\cancel{(e^3-1)}}\right]$

$$\int_2^6 x^2\sqrt{x-1}\,dx = \int x^2\sqrt{u}\cdot du = \int (u^2+2u+1)\cdot u^{\frac{1}{2}}\,du$$

$$\int \left(u^{5/2}+2u^{3/2}+1u^{\frac{1}{2}}\right)du$$

$u = x-1 \Rightarrow u+1 = x$

$du = dx \quad x^2 = (u+1)^2 = (u+1)(u+1) = u^2+2u+1$

$$\frac{2}{7}u^{5/2+1=\frac{7}{2}} + 2\cdot\frac{2}{5}\cdot u^{3/2+1=\frac{5}{2}} + \frac{2}{3}u^{\frac{1}{2}+1=\frac{3}{2}}$$

$$\frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2}\Big[_2^6$$

$6-1=5$

$2-1=1$

$$\frac{2}{7}(5)^{7/2} + \frac{4}{5}(5)^{5/2} + \frac{2}{3}(5)^{3/2} - \left[\frac{2}{7}\cdot 1^{7/2} + \frac{4}{5}\cdot 1^{5/2} + \frac{2}{3}(1)^{3/2}\right]$$

$$5^{\frac{1}{2}}\left[\frac{2}{7}\cdot 5^3 + \frac{4}{5}\cdot 5^2 + \frac{2}{3}\cdot 5\right] - \left[\frac{2}{7} + \frac{4}{5} + \frac{2}{3}\right]$$

$$5^{\frac{1}{2}}\left[\frac{250}{7} + \frac{100}{5} + \frac{10}{3}\right] - \left[\frac{2}{7} + \frac{4}{5} + \frac{2}{3}\right]$$

$$\frac{1240\sqrt{5}}{21} - \frac{184}{105}$$

$$\int_2^6 x^2\sqrt{x+3}\,dx = \int_5^9 (u-3)^2\cdot u^{\frac{1}{2}}\,du = \int_5^9 (u^2-6u+9)\,u^{\frac{1}{2}}\,du$$

$u = x+3 \Rightarrow u-3 = x$

$du = dx$

$u = 6+3 = 9$

$u = 2+3 = 5$

$$\int_5^9 \left(u^{5/2} - 6u^{\frac{3}{2}} + 9u^{\frac{1}{2}}\right)du$$

$$\frac{2}{7}u^{5/2+1=\frac{7}{2}} - 6\cdot\frac{2}{5}\cdot u^{\frac{3}{2}+1=\frac{5}{2}} + 9\cdot\frac{2}{3}\cdot u^{\frac{1}{2}+1=\frac{3}{2}}$$

$$\frac{2}{7}u^{7/2} - \frac{12}{5}u^{5/2} + 6u^{3/2}\Big[_5^9 = \left[\frac{2}{7}\cdot 3^7 - \frac{12}{5}\cdot 3^5 + 6\cdot 3^3\right] - \left[\frac{2}{7}\cdot 5^{7/2} - \frac{12}{5}\cdot 5^{5/2} + 6\cdot 5^{3/2}\right]$$

$$203\,\frac{23}{35} - \sqrt{5}\left(5\,\frac{5}{7}\right]$$

$$9^{7/2} = (3^2)^{7/2} = 3^{\cancel{14/2}} = 3^7$$

3. $\int_0^1 x^3 e^{x^4+1}\,dx = \int_1^2 \cancel{x^3} e^u \cdot \frac{du}{4\cancel{x^3}} = \frac{1}{4}\int_1^2 e^u\,du = \frac{1}{4}e^u + c$

$u = x^4+1$

$du = 4x^3\,dx$

$\frac{du}{4x^3} = dx$

$\frac{1}{4}e^{x^4+1} + c \Big|_0^1 = \frac{1}{4}e^{1^4+1} - \frac{1}{4}e^{0^4+1}$

$\boxed{\frac{1}{4}e^2 - \frac{1}{4}e^1 = \frac{1}{4}e(e-1)}$

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3. $\int_0^1 x^2 e^{x^3+1}\,dx = \int_1^2 \cancel{x^2} e^u \cdot \frac{du}{3\cancel{x^2}} = \frac{1}{3}\int_1^2 e^u\,du = \frac{1}{3}e^u + c \Big|_1^2$

$u = x^3+1$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

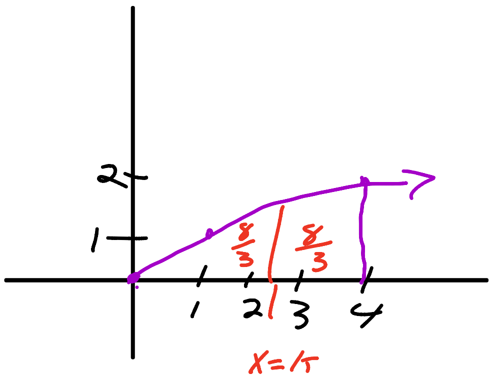
$\frac{1}{3}e^2 - \frac{1}{3}e^1$

$\frac{e}{3}(e^1 - 1)$

$1^3+1=2$

$0^3+1=1$

7. The vertical line $x = k$ divides the region enclosed by the graphs of $y = \sqrt{x}$, $y = 0$, and $x = 4$ into 2 regions of equal area. Find the value of k.



$\int_0^4 x^{\frac{1}{2}} dx = \frac{2}{3} \cdot x^{\frac{3}{2}} \Big|_0^4$

$\frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2}$

$\frac{2}{3}(8) - 0 = \frac{16}{3}$

$\frac{1}{2} \cdot \frac{16}{3} = \frac{8}{3}$

$\int_0^k x^{\frac{1}{2}} dx = \frac{8}{3}$

$\frac{2}{3}(k)^{3/2} - 0 = \frac{8}{3}$

$\cancel{\frac{3}{2}} \cdot \frac{2}{3} k^{3/2} = \frac{8}{\cancel{3}} \cdot \frac{\cancel{3}}{2}$

$(k^{3/2})^{\frac{2}{3}} = (4)^{2/3}$

$k^{\cancel{\frac{3}{2}\cdot\frac{2}{3}}} = 2^{2 \cdot \frac{2}{3}}$

$k = 2^{4/3} = 2\sqrt[3]{2}$

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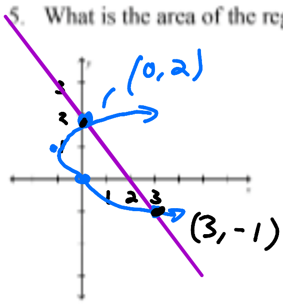
5. What is the area of the region bounded by the graphs of $x = y^2 - 2y$ and $y = -x + 2$?



$x = y(y-2)$

$x = y^2 - 2y \qquad x = 2 - y$

$y^2 - 2y = 2 - y$

$+y \;\; -2 \quad -2 + y$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2$ or $y = -1$

$(-1)^2 - 2(-1) = 1 + 2$

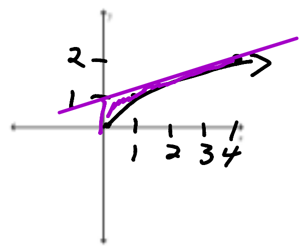
$\int_{-1}^{2} \left[(2-y) - (y^2 - 2y) \right] dy$

$\int_{-1}^{2} \left[-y^2 + y + 2 \right] dy$

$-\frac{1}{3}y^3 + \frac{1}{2}y^2 + 2y \Big|_{-1}^{2}$

$-\frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 + 2(2) - \left[-\frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2 + 2(-1) \right] = -\frac{8}{3} + 2 + 4 - \frac{1}{3} - \frac{1}{2} + 2$

4. Find the area enclosed by the curve $y = \sqrt{x}$, the *tangent* to the curve at $x = 4$, and the $y - axis$.



$y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

at $x = 4$

$\frac{1}{2\sqrt{4}} = \frac{1}{2\cdot 2} = \frac{1}{4}$ slope

Point $(4,2)$

Tangent

$y = mx + b$

$y = \frac{1}{4}x + b$

$2 = \frac{1}{4}(4) + b$

$2 = 1 + b$

$1 = b$

$y = \frac{1}{4}x + 1$

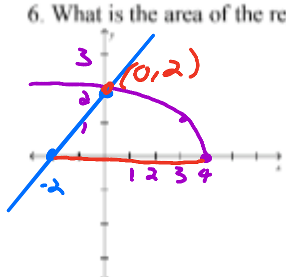
$$\int_0^4 \left[\left(\frac{1}{4}x+1\right) - \sqrt{x}\right]dx = \int_0^4 \left[\frac{1}{4}x + 1 - x^{\frac{1}{2}}\right]dx$$

$$\frac{1}{4}\cdot\frac{1}{2}\cdot x^{1+1=2} + x - \frac{2}{3}x^{\frac{1}{2}+1=\frac{3}{2}} \Big|_0^4$$

$$\frac{1}{8}(4)^2 + 4 - \frac{2}{3}(4)^{3/2} - \left[\cancel{\frac{1}{8}(0)^2 + 0 - \frac{2}{3}(0)^{3/2}}\right]$$

$$\frac{16}{8} + 4 - \frac{2\cdot 8}{3} - 0$$

6. What is the area of the region bounded by the graphs of $f(x) = \sqrt{4 - x}$ and $g(x) = x + 2$ and the x-axis?



$$\int_{-2}^{0} (x+2)dx + \int_0^4 (4-x)^{\frac{1}{2}}dx$$

$y = \sqrt{4-x}$ $\quad$ $y = x+2$

$x = 4 - y^2$ $\quad$ $x = y - 2$

$$\int_0^2 [(4-y^2) - (y-2)]dy$$

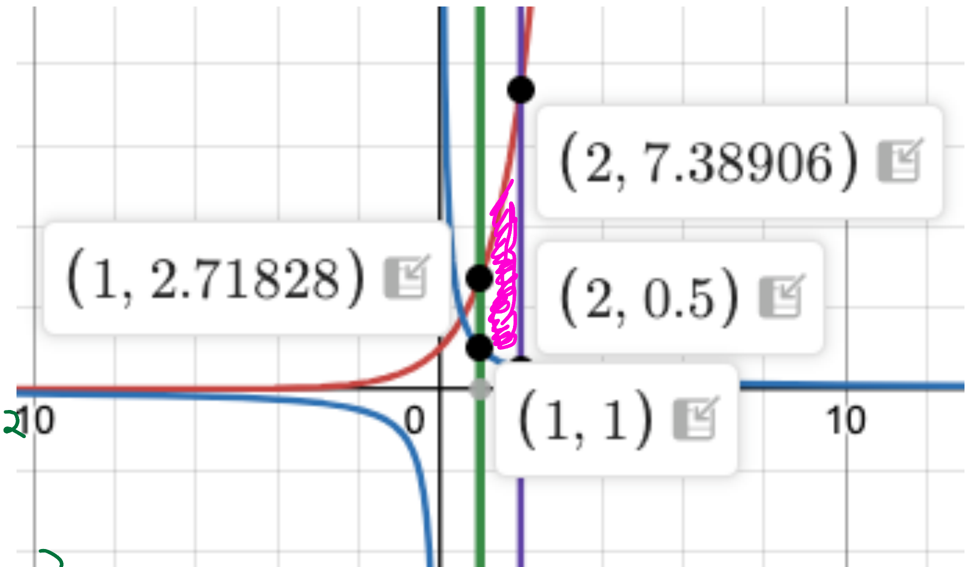
$$\int_0^2 [6 - y^2 - y]dy = 6y - \frac{1}{3}y^3 - \frac{1}{2}y^2 \Big|_0^2$$

$$6(2) - \frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 - \left[\cancel{6(0) - \frac{1}{3}(0)^3 - \frac{1}{2}(0)^2}\right]$$

$$12 - \frac{8}{3} - 2 = 12 - 2\frac{2}{3} - 2 = 7\frac{1}{3}$$

Calculator May be used #3-7. Please Show Set-up.

3. Find the area of the region bounded by the graphs of $y = e^x, y = \frac{1}{x}, x = 1, and\ x = 2$



$$\int_1^2 (e^x - \tfrac{1}{x})dx$$

$$e^x - \ln|x| + c \Big[_1^2$$

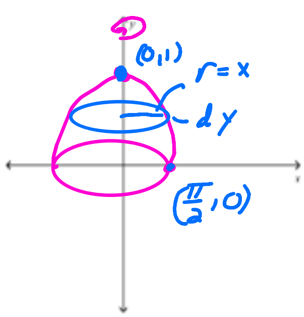
$$e^2 - \ln 2 - [e^1 - \ln 1]$$

$$e^2 - \ln 2 - e + 0$$

$$\int_1^2 \left(e^x - \frac{1}{(x)}\right)dx$$

$= 3.97762708991$

6. What is the volume of the solid of revolution generated when the region in the first quadrant bounded by $y = \cos x$ is revolved about the y-axis.



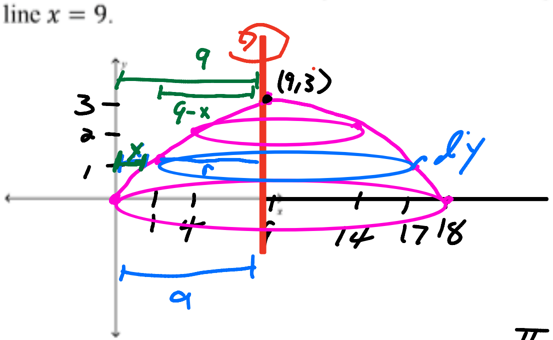
$$\int_0^1 \pi (x)^2 \cdot dy$$

$y = \cos x$

$x = \cos^{-1} y$

$$\int_0^1 \pi (\arccos y)^2 dy = \pi(\pi - 2)$$

3. Find the volume of the solid generated when the region bound by $y = \sqrt{x}$, $y = 0$, $x = 9$ is revolved about the line $x = 9$.



$r = 9 - x$

$y^2 = x$

$\sqrt{9} = 3$

$$\int_0^3 \pi(9-x)^2\,dy$$

$$\pi\int_0^3 (81 - 18x + x^2)\,dx$$

$$\pi\int_0^3 (81 - 18y^2 + y^4)\,dy$$

$$\pi\left[81y - 6y^3 + \tfrac{1}{5}y^5\right]\Big|_0^3 =$$

$$\pi\left[81(3) - 6(3)^3 + \tfrac{1}{5}(3)^5\right] - \pi\left[\cancel{81(0) - 6(0)^3 + \tfrac{1}{5}(0)^5}\right]$$

$$\pi\left[243 - 162 + \frac{243}{5}\right]$$

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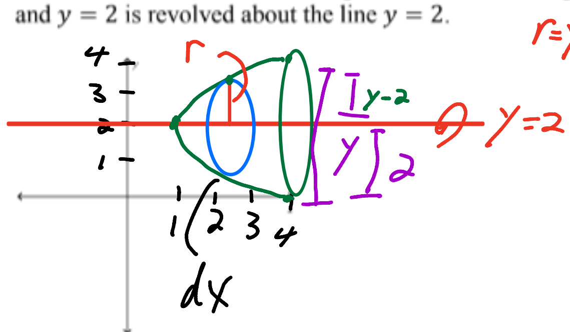
4. Find the volume of the solid generated when the region bound in the first quadrant by $y = 2\sqrt{x}$, $x = 1$, $x = 4$, and $y = 2$ is revolved about the line $y = 2$.

$r = y - 2$



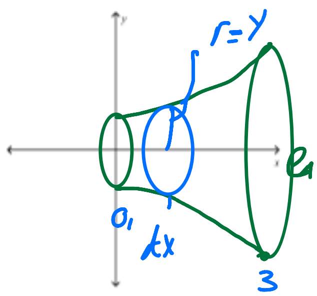
$$\int_1^4 \pi(y-2)^2\,dx$$

$$\int_1^4 \pi(2\sqrt{x} - 2)^2\,dx$$

$$\pi\int_1^4 (4x - 8\sqrt{x} + 4)\,dx = \pi\left[2x^2 - \tfrac{16}{3}x^{3/2} + 4x\right]\Big|_1^4$$

$$\pi\left[\left(2\cdot 4^2 - \tfrac{16}{3}(4)^{3/2} + 4\cdot 4\right) - \left(2\cdot 1^2 - \tfrac{16}{3}(1)^{3/2} + 4(1)\right)\right]$$

5. *for this question, do not use a calculator to evaluate the integral; do it by hand, but simplify as much as possible.*
What is the volume of the solid of revolution generated when the region in the first quadrant bounded by the graph of $y = e^x$, the x-axis, and the line $x = 3$ is revolved about the x-axis.



$$\int_0^3 \pi r^2 dx$$

$$\pi \int_0^3 (e^x)^2 dx$$

$$\pi \int_0^3 e^{2x} dx = \pi \int e^u \cdot \frac{du}{2} = \frac{1}{2} e^u + c$$

$u = 2x$
$du = 2dx$
$\frac{du}{2} = dx$

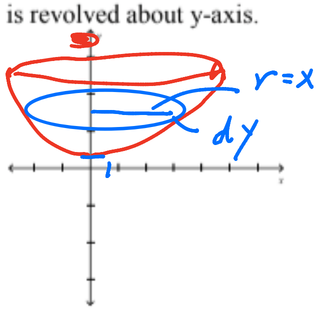
$$\pi\left(\frac{1}{2} e^{2x} + c\right)\Big|_0^3$$

$$\pi\left(\frac{1}{2} \cdot e^{2 \cdot 3} - \frac{1}{2} e^{2 \cdot 0}\right) = \left(\frac{1}{2} e^6 - \frac{1}{2} e^0\right)\pi$$

$$\left(\frac{e^6}{2} - \frac{1}{2}\right)\pi$$

2. Find the volume of the solid that results when the region enclosed by the y-axis, $y = x^3 + 1$, and $y = 9$. is revolved about y-axis.

$$\sqrt[3]{y-1} = x$$



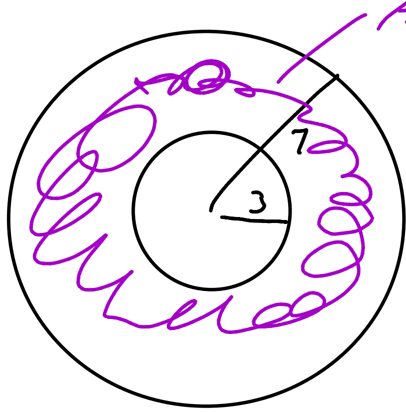
$$\int_1^9 \pi (x)^2 dy = \int_1^9 \pi \left(\sqrt[3]{y-1}\right)^2 dy$$

$$\pi \int_1^9 (y-1)^{\frac{2}{3}} dy \Rightarrow \pi \int u^{\frac{2}{3}} \cdot du = \frac{3}{5} u^{\frac{2}{3}+1 = \frac{5}{3}}$$

$u = y-1$
$du = dx$

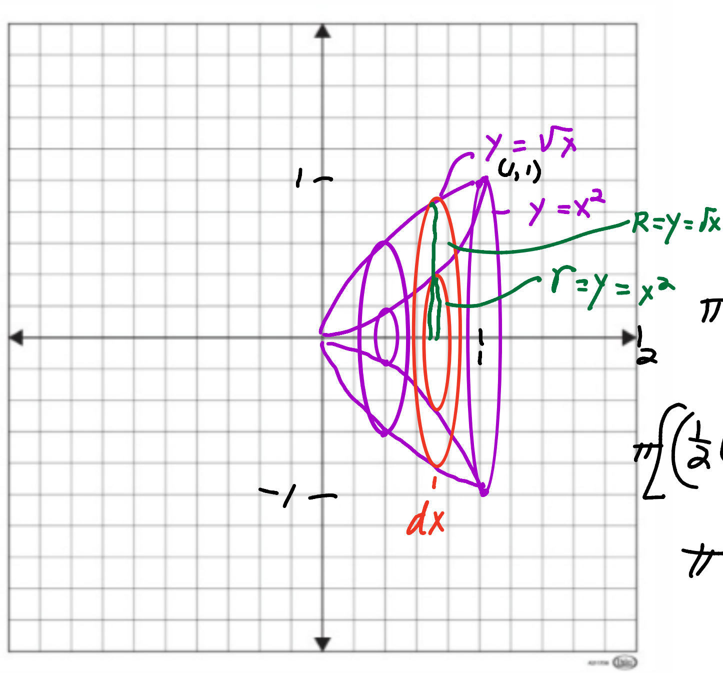
$$\pi (y-1)^{\frac{5}{3}} \cdot \frac{3}{5} \Big|_1^9$$

Area $= \pi(7)^2 - \pi(3)^2 = \pi\cdot 49 - 9\pi$



40π

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ About the x-axis



$$\int_0^1 \pi(R^2 - r^2)\,dx$$

$$\pi\int_0^1 \left[(\sqrt{x})^2 - (x^2)^2\right]dx$$

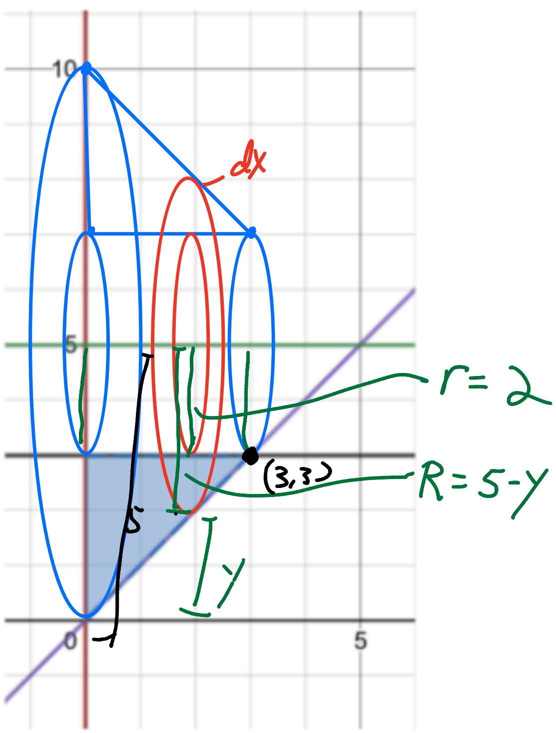
$$\pi\int_0^1 (x - x^4)\,dx = \pi\left(\tfrac{1}{2}x^2 - \tfrac{1}{5}x^5\right)\Big|_0^1$$

$$\pi\left[\left(\tfrac{1}{2}(1)^2 - \tfrac{1}{5}(1)^5\right) - \left(\cancel{\tfrac{1}{2}(0)^2 - \tfrac{1}{5}(0)^5}\right)\right]$$

$$\pi\left(\tfrac{1}{2} - \tfrac{1}{5}\right) = \pi\cdot\tfrac{3}{10}$$

Example 3

Find the volume of the solid generated by re
region bounded by the graphs of $y = x$, $y = 3$,
about the line $y = 5$.



$$\int_0^3 \pi\left[(5-y)^2 - (2)^2\right]dx$$

$$\pi\int_0^3 \left[(5-x)^2 - 4\right]dx$$

$$\pi\int_0^3 \left[25 - 10x + x^2 - 4\right]dx$$

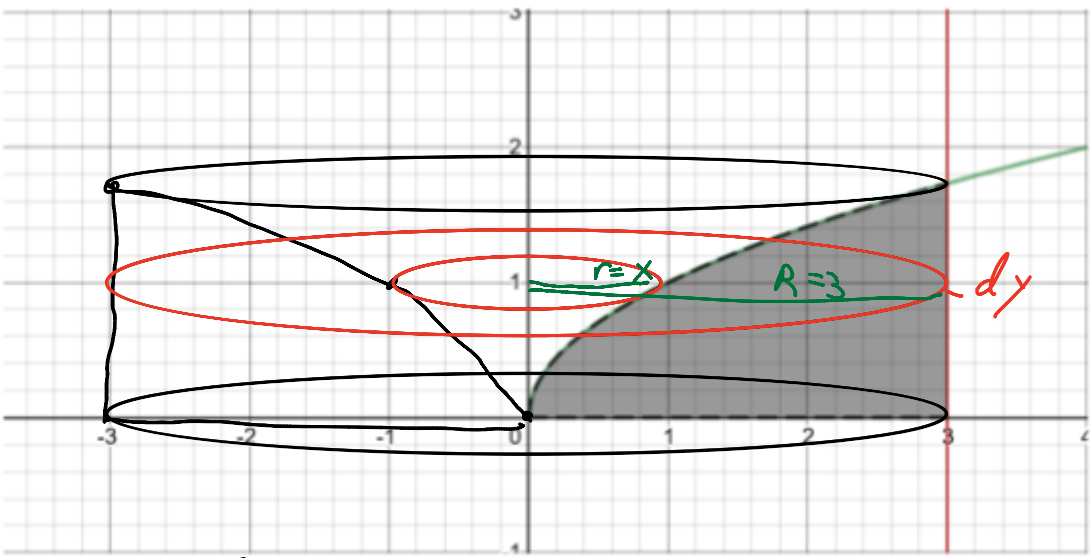
$$\pi\int_0^3 \left(21 - 10x + x^2\right)dx$$

$$\pi\left[21x - 5x^2 + \frac{1}{3}x^3\right]\Big|_0^3$$

Keep going

Example 4

Find the volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x}$, $y = 0$, $x = 3$, rotated about the y-axis. $y^2 = x$



$$\int_0^{\sqrt{3}} \pi(3^2 - x^2)\,dy = \int_0^{\sqrt{3}} \pi(9 - (y^2)^2)\,dy = \int_0^{\sqrt{3}} \pi(9 - y^4)\,dy$$

$$\pi\left[9y - \frac{1}{5}y^5\right]\Big|_0^{\sqrt{3}}$$

Example 5

Find the volume of the solid generated by revolving the region bounded by the graphs of $y = 4$, $y = x^2$, x=0 about the line $x = -2$